

**QUANTUM CHAOS AND NON-LINEAR STATISTICAL MECHANICS OF COMPLEX BIS PROCESSES IN THE FOUR DIMENSIONAL WORLD****Deepak Kumar\*<sup>@</sup>, Tuhin Dutta\*, Madan Mohan Bajaj\*\*\* and Sobinder Singh<sup>#</sup>**

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<sup>#</sup>Department of Applied Sciences, Maharaja Surajmal Institute of Technology, Guru Gobind Singh Indraprastha University, Delhi, India.(E-mail: \*<sup>@</sup>[deepakm256@gmail.com](mailto:deepakm256@gmail.com); \*\*[tuhindutta38@gmail.com](mailto:tuhindutta38@gmail.com); \*\*\*[madanmohanbajaj@rediffmail.com](mailto:madanmohanbajaj@rediffmail.com);\*\*\*\*[sobinder\\_singh@yahoo.co.in](mailto:sobinder_singh@yahoo.co.in))**ABSTRACT**

BIS means breakdown of integrated system. EP waves are generated due to the annihilation of living creatures. BIS effect leads to BIS manifold. In simple words, killing of living creatures leads to unexpected serious catastrophes and chaotic situations everywhere. As a consequence the topology of the universe changes. These consequences, we represent by the symbol M and we pronounce it as the BIS manifold. BIS manifold is the result of the perturbations, disturbances and destructions created by weak, moderate, strong and extremely strong BIS processes. In our present research paper we will study the statistical mechanics of the complex BIS processes leading to quantum chaos.

**KEY WORDS:** breakdown of integrated system, EP waves, statistical mechanics.**1 Origin of BIS manifold:**

We will now show how the n-dimensional BIS manifold originates.

Suppose n = number of dimensions in which we are studying the impact of BIS processes

 $m^n$  = n-dimensional complete minimal immersed submanifold of a complex BIS process in n-dimensional. Euclidean space  $\square^{n+p}$ 

$$\int_m (|\nabla f|^2 - |A|^2 f^2) \geq 0, \forall f \in C_0^\alpha(m) \quad (1)$$

(All values of the function f are elements of the set  $C_0^\alpha$  of the BIS manifold M) $\int_m$  = Integral for the BIS manifold $\square$  = Euclidean space $\square^{n+p}$  = (n+p)-dimensional Euclidean space

n+p = Total number of dimensions.

 $m^n$  (n ≥ 3) = an n-dimensional complete immersed $\frac{n-2}{n}$  super stable minimal submanifold in an (n+p)-dimensional Euclidean**2 Rigidity of minimal submanifold of BIS processes**

In the present work, we shall study the rigidity of minimal BIS submanifolds with flat normal bundle of complex BIS processes. If the second fundamental form of M satisfied some decay conditions, then M is an affine plane or a catenoid in some Euclidean subspace.

**3 BIS Processes are very strong. EP Waves penetrate in 4π Geometry**

In one or two dimensions, we see their consequences immediately with naked eyes. Only we have to have open and analytic brain and we should not close down our eyes to the fundamental laws of physics (Newton's third law of motion etc.). Impact of BIS processes in 3-dimensions can also be realized by examining the vertical scale. But consequences of BIS processes in n-dimensions are more pronounced.

**4 BIS processes in Regional wars and world wars**Wars tells us, what BIS process do in 4-dimensional (x,y,z ict). Let us consider a number  $\delta$ . If

$$0 \leq \int_m (|\nabla f|^2 - |A|^2 f^2) \geq 0, \forall f \in C_0^\alpha(m)$$

If  $\delta_1 > \delta_2$ ,  $\delta_1$ -Stable implies,  $\delta_2$ -Stable

Therefore M is stable means M is  $\delta$ -stable.

Two conditions, when BIS manifold becomes hyperplane

(i) M is a stable complete minimal hypersurface in  $(n+1)$ -dimensional Euclidean space,  $\square^{n+p}$

$$(ii) \quad \lim_{R \rightarrow \infty} \frac{1}{R^{2+2q}} \int_{\substack{B(2R) \\ B(R)}} |A|^2 = 0, \quad q < \sqrt{\frac{2}{n}}$$

$\delta$ -super-stable minimal submanifolds in  $\square^{n+p}$  with flat normal bundle.

Three situations when BIS manifold becomes An Affine Plane

Let  $M^n (n \geq 3)$  be a  $\delta$ -super-stable complete immersed minimal submanifold in  $\square^{n+p}$  with flat normal bundle of BIS processes.

(i) If M is  $\delta \left( > \frac{n-2}{n} \right)$ -super-stable and

$$\lim_{R \rightarrow \infty} \frac{1}{R^{2+2q}} \delta \int_{\substack{B(2R) \\ B(R)}} |A|^{2\delta} = 0, \quad q < \sqrt{\left( \frac{2}{n} - 1 \right)} \delta^{-1} + 1$$

then M = an affine plane.

(ii) If M is  $\delta \left( > \frac{n-2}{n} \right)$ -super-stable and

$$\lim_{R \rightarrow \infty} \frac{1}{R^2} \int_{\substack{B(2R) \\ B(R)}} |A|^{\frac{2(n-2)}{n}} = 0,$$

M = a catenoid in some Euclidean subspace = an affine plane.

Let  $M^n$  be an n-dimensional minimal immersed submanifold in  $\square^{n+p}$ . We choose an orthonormal frame  $e_1, e_2, \dots, e_{n+p}$  in  $\square^{n+p}$  such that, restricted to M, the vectors  $e_1, e_2, \dots, e_n$  are tangent to M. And we shall denote the second fundamental form by  $h_{ij}^\alpha$ . Then we have  $|A|^2 = \sum (h_{ij}^\alpha)^2$  and

$$2|A| \Delta |A| + 2|\nabla |A||^2 = \Delta |A|^2 = 2 \sum (h_{ij}^\alpha)^2 + 2 \sum (h_{ij}^\alpha) \Delta h_{ij}^\alpha \quad (2)$$

By eq. (3) and (2), we have

$$\sum (h_{ij}^\alpha) \Delta h_{ij}^\alpha = - \sum (h_{ik}^\alpha h_{jk}^\beta - h_{jk}^\alpha h_{ik}^\beta) (h_{il}^\alpha h_{jl}^\beta - h_{jl}^\alpha h_{il}^\beta) - \sum h_{ij}^\alpha h_{kl}^\alpha h_{ij}^\beta h_{kl}^\beta \quad (3)$$

Let M be an n-dimensional immersed minimal BIS submanifold in  $\square^{n+p}$  with flat  $(n+p)$ -dimensional Euclidean manifold  $\square =$ Euclidean manifold  $(n+p) =$  total no. of dimensions.

$$|A| \Delta |A| + |\nabla |A||^2 \geq \frac{2}{n} |\nabla |A||^2 + E \quad (4)$$

where  $E = \sum_{\alpha \neq \beta} (\sum_{i,j} (h_{ij}^\alpha)^2) (\sum_{i,j} (h_{ij}^\beta)^2)$  is nonnegative

Since M has flat normal bundle, we have  $h_{ik}^\alpha h_{jk}^\beta - h_{jk}^\alpha h_{ik}^\beta = 0$ . Therefore, from (2) we obtain

$$\sum (h_{ij}^\alpha) \Delta h_{ij}^\alpha = - \sum h_{ij}^\alpha h_{kl}^\alpha h_{ij}^\beta h_{kl}^\beta$$

For each  $\alpha$   $H_\alpha$  the symmetric matrix  $(h_{ij}^\alpha)$ , and  $S_{\alpha\beta} = \sum h_{ij}^\alpha h_{ij}^\beta$ . Then the  $(p \times p)$  matrix  $(S_{\alpha\beta})$  is symmetric. It can be assumed to be diagonal for a suitable choice of  $e_{n+1}, \dots, e_{n+p}$ . Thus we have from

$$\sum (h_{ij}^\alpha) \Delta h_{ij}^\alpha = - \sum S_{\alpha\alpha}^2 = - \sum_\alpha \left( \sum_{i,j} (h_{ij}^\alpha)^2 \right)^2 \quad (5)$$

on the other hand,

$$|A|^4 = (|A|^2)^2 = \left( \sum_{\alpha} \sum_{ij} (h_{ij}^{\alpha})^2 \right)^2 = \sum_{\alpha} \left( \sum_{ij} (h_{ij}^{\alpha})^2 \right)^2 + E \quad (6)$$

Hence from (1), (5) and (6) we have

$$2|A|\Delta|A| + 2|\nabla|A||^2 = 2\sum (h_{ijk}^{\alpha})^2 + 2E - 2|A|^4 \quad (7)$$

Since  $\sum (h_{ijk}^{\alpha})^2 = |\Delta A|^2$ , from the above equality we get

$$\begin{aligned} |A|\Delta|A| + |\nabla|A||^2 &= |\nabla A|^2 + E - |A|^4 \\ \therefore |\Delta|A||^2 - |\nabla A|^2 &= E - |A|^4 - |A|\Delta|A| \end{aligned}$$

For M, Xin [13] gave the following estimate:

$$|\nabla A|^2 - |\Delta|A||^2 \geq \frac{2}{n} |\nabla|A||^2$$

Combining with (7), we obtain

$$|A|\Delta|A| + |A|^4 \geq \frac{2}{n} |\nabla|A||^2 + E$$

Proof of Theorem 1.1 for any  $\epsilon > 0$  and  $a > 0$ , let  $u = (|A|^2 + \epsilon)^{\frac{a}{2}}$ , then

$$\begin{aligned} \Delta u &= u(\Delta \log u + |\nabla \log u|^2) \\ &= \frac{au}{2} \left( \frac{\Delta|A|^2}{|A|^2 + \epsilon} - \frac{|\nabla|A||^2}{(|A|^2 + \epsilon)^2} \right) + \frac{ua^2|\nabla|A||^2}{4(|A|^2 + \epsilon)^2} \\ &= au \left( \frac{1/2 \Delta|A|^2}{|A|^2 + \epsilon} + (a-2) \frac{|A|^2|\nabla|A||^2}{(|A|^2 + \epsilon)^2} \right) \\ &\geq au \left( \frac{\left(1 + \frac{2}{n}\right) \Delta|A|^2 - |A|^4 + E}{|A|^2 + \epsilon} + (a-2) \frac{|A|^2|\nabla|A||^2}{(|A|^2 + \epsilon)^2} \right) \\ &= -au \frac{(|A|^2 + \epsilon)^2}{|A|^2 + \epsilon} + au \frac{2\epsilon|A|^2}{|A|^2 + \epsilon} + au \frac{2\epsilon^2}{|A|^2 + \epsilon} + au \frac{E}{|A|^2 + \epsilon} \\ &\quad + au \left( \frac{\left(1 + \frac{2}{n}\right) (|A|^2 + \epsilon) |\Delta|A||^2 - (a-2) |A|^2 |\nabla|A||^2}{(|A|^2 + \epsilon)^2} \right) \\ &= -au(|A|^2 + \epsilon) + au \frac{E}{|A|^2 + \epsilon} + \left( \frac{2}{n} + a - 1 \right) au \frac{|A|^2 |\nabla|A||^2}{(|A|^2 + \epsilon)^2} \\ &= +au \frac{2\epsilon|A|^2}{|A|^2 + \epsilon} + au \frac{2\epsilon^2}{|A|^2 + \epsilon} + au \frac{\left(1 + \frac{2}{n}\right) \epsilon |\nabla|A||^2}{(|A|^2 + \epsilon)^2} \end{aligned}$$

$$\geq -au(|A|^2 + \varepsilon) + au \frac{E}{|A|^2 + \varepsilon} + \left(\frac{2}{n} + a - 1\right) au \frac{|A|^2 |\nabla |A||^2}{(|A|^2 + \varepsilon)^2} \quad (8)$$

Where we apply (3) in Lemma 1. It follows from (8) by directly computing that

$$u\Delta u \geq -au^{2+\frac{2}{a}} + au^{2-\frac{2}{a}}E + \left(\frac{2}{n} + a - 1\right)a^{-1} |\nabla u|^2 \quad (9)$$

Let  $q \geq 0$  and  $f \in C_0^\infty(M)$ . Multiplying (9) by  $u^{2q}f^2$  and integrating over  $M$ , we obtain

$$\begin{aligned} & a \int_M u^{2(1+q)-\frac{2}{a}} f^2 E + \left(\frac{2}{n} + a - 1\right) a^{-1} \int_M |\nabla u|^2 u^{2q} f^2 \\ & \leq a \int_M u^{2(1+q)+\frac{2}{a}} f^2 + \int_M u^{2q+1} f^2 \Delta u \\ & = a \int_M u^{2(1+q)+\frac{2}{a}} f^2 - 2 \int_M u^{2q+1} f \langle \Delta f, \nabla u \rangle \\ & \quad - (2q+1) \int_M u^{2q} f^2 |\Delta u|^2, \end{aligned}$$

Which gives

$$\begin{aligned} & a \int_M u^{2(1+q)+\frac{2}{a}} f^2 E + \left(2(q+1) - \frac{n-2}{na}\right) \int_M |\Delta u|^2 u^{2q} f^2 \\ & \leq a \int_M u^{2(1+q)+\frac{2}{a}} f^2 - 2 \int_M u^{2q+1} f \langle \nabla f, \nabla u \rangle \quad (10) \end{aligned}$$

Using the Cauchy-Schwarz inequality, we can rewrite (10) as

$$\left(2(q+1) - \frac{n-2}{na} - \varepsilon'\right) \int_M u^{2q} f^2 |\nabla u|^2 + a \int_M u^{2(1+q)-\frac{2}{a}} f^2 E \quad (11)$$

On the other hand, replacing  $f$  by  $u^{(1+q)}f$  in the  $\delta$ -super-stability inequality (4), we have

$$\begin{aligned} & \delta \int_M \left(u^{\frac{2}{a}-\varepsilon}\right) u^{2(1+q)} f^2 \leq \delta \int_M |A|^2 u^{2(1+q)} f^2 \\ & \delta \int_M \left(u^{\frac{2}{a}-\varepsilon}\right) u^{2(1+q)} f^2 \leq \delta \int_M |A|^2 u^{2(1+q)} f^2 \\ & \leq 2(1+q) \int_M u^{(2q+1)} f \langle \nabla f, \nabla u \rangle \\ & \quad \therefore \delta \int_M \left(u^{\frac{2}{a}-\varepsilon}\right) u^{2(1+q)} f^2 \\ & \leq (1+q)(1+q+\varepsilon') \int_M u^{2q} f^2 |\nabla u|^2 + \left(1 + \frac{1+q}{\varepsilon'}\right) \int_M u^{2(1+q)} |\nabla f|^2 \quad (12) \end{aligned}$$

(1) When  $\delta > \frac{n-2}{n}$  If  $2(q+1) - \frac{n-2}{na} \varepsilon' - > 0$  then introducing (11) to (13), we obtain

$$\begin{aligned} & \left[ \left( 2(q+1) - \frac{n-2}{na} - \varepsilon' \right) \delta - (1+q)(1+q+\varepsilon')a \right] \int_M u^{2(1+q)+\frac{2}{a}} f^2 \\ & \leq \left[ \left( 2(q+1) - \frac{n-2}{na} - \varepsilon' \right) \left( 1 + \frac{1+q}{\varepsilon'} \right) + \frac{(1+q)(1+q+\varepsilon')}{\varepsilon'} \right] \\ & \quad x \int_M u^{2(1+q)} |\nabla f|^2 + \varepsilon \left( 2(q+1) - \frac{n-2}{na} - \varepsilon' \right) \delta \int_M u^{2(1+q)} f^2 \quad (13) \end{aligned}$$

(i) If  $a = \delta$ , taking  $q < \sqrt{\left(\frac{2}{n}-1\right)\delta^{-1}} + 1$ , we see that

$$\left( 2(q+1) - \frac{n-2}{n\delta} \right) \delta - (1+q)^2 \delta > 0, \quad (14)$$

and then we can choose  $\varepsilon > 0$  sufficiently small so that

$$\left( 2(q+1) - \frac{n-2}{n\delta} - \varepsilon' \right) \delta - (1+q)(1+q+\varepsilon')\delta > 0,$$

Let  $\varepsilon \rightarrow 0$ , it follows from (14) that for  $\sqrt{\left(\frac{2}{n}-1\right)\delta^{-1}} + 1$  the following inequality holds:

$$\int_M u^{2(1+q)+\frac{2}{\delta}} f^2 \leq C_1 \int_M u^{2(1+q)} |\nabla f|^2, \quad (15)$$

where  $C_1$  is a constant that depends on  $n$ ,  $\varepsilon'$  and  $q$ . We now try to transform (15) the right hand side involved  $u$  in the power two. For that, we use Young's inequality:

$$ab \leq \frac{\beta^s a^s}{s} + \frac{\beta^{-t} b^t}{t}, \frac{1}{s} + \frac{1}{t} = 1, \quad (16)$$

where  $\beta > 0$  is arbitrary and  $1 < s < \infty, 1 < t < \infty$ . Let  $r, 0 < r < 2+2q$ , be a number yet to be determined. By using (16) we obtain

$$\begin{aligned} u^{2+2q} |\nabla f|^2 &= f^2 \left( u^{2+2q} \frac{|\nabla f|^2}{f^2} \right) = f^2 \left( u^{2+2q-r} u^r \frac{|\nabla f|^2}{f^2} \right) \\ &\leq f^2 \left( \frac{\beta^s}{s} u^{s(2+2q-r)} + \frac{\beta^{-t}}{t} \left( u^r \frac{|\nabla f|^2}{f^2} \right)^t \right) \end{aligned} \quad (17)$$

We now choose  $r$  to satisfy the following equations:

$$s(2+2q-r) = 2+2q + \frac{2}{\delta}, \quad rt = 2, \quad \frac{1}{s} + \frac{1}{t} = 1$$

This is indeed possible, and the solution is

$$r = \frac{2}{1+q\delta}, \quad t = 1+q\delta, \quad s = \frac{1+q\delta}{q\delta}$$

Using these values and the fact the  $\beta$  may be made small, from (15) and (17) we obtain

$$\int_M u^{2+2q+\frac{2}{\delta}} f^2 \leq C_2 \int_M u^{\frac{2|\nabla f|^{2+2q\delta}}{f^2 q^\delta}} \quad (18)$$

Where  $C_2$  is a constant that depends on  $n, \epsilon', \beta$  and  $q$ . Now we use the arbitrariness of  $f$  to replace  $f$  by  $f^{1+q\delta}$  in (18) and obtain

$$\int_M u^{2+2q+\frac{2}{\delta}} f^{2+2q\delta} \leq C_3 \int_M u^{2|\nabla f|^2+2q\delta} \tag{19}$$

Let  $f$  be a smooth function on  $(0, \infty)$  such that  $f \geq 0$ ,  $f=1$  on  $[0, R]$  and  $f=0$  in  $[2R, \infty]$  with  $|f'| \leq \frac{2}{R}$ . Then considering  $f$  on  $r$ , where  $r$  is the function in the definition of  $B(R)$ , we have from (19)

$$\int_{B(R)} u^{2+2q+\frac{2}{\delta}} \leq \frac{4C_3}{R^{2+2q\delta}} \int_{B(2R) \setminus B(R)} u^2 \tag{20}$$

Let  $R \rightarrow +\infty$  by assumption that  $\lim_{R \rightarrow \infty} \frac{1}{R^{2+2q\delta}} \int_{B(2R) \setminus B(R)} |A|^{2\delta} = 0$ , from (20) we conclude  $|A|=0$ , i.e.,  $M$  is an affine plane.

(ii) If  $a = \frac{n-2}{n}$ , taking  $q=0$ , we see that

$$\left( 2(q+1) - \frac{n-2}{na} \right) \delta - (1+q)^2 a > 0,$$

and then we can choose  $\epsilon' > 0$  sufficiently small so that

$$\left( 2(q+1) - \frac{n-2}{na} - \epsilon' \right) \delta - (1+q)(1+q+\epsilon')a > 0, \tag{21}$$

Let  $\epsilon \rightarrow 0$ , it follows from (14) that the following inequality holds:

$$\int_M u^{2+\frac{2n}{n-2}} f^2 \leq C_4 \int_M u^{2|\nabla f|^2}$$

where  $C_4$  is a constant that depends on  $n, \epsilon'$ . Under the assumption that  $\lim_{R \rightarrow +\infty} \frac{1}{R^2} \int_{B(2R) \setminus B(R)} |A|^{\frac{2(n-2)}{n}} = 0$ , from

(21) we conclude that  $|A|=0$ , i.e.,  $M$  is an affine

(iii) When  $\delta = \frac{n-2}{n}$

Multiplying (10) by  $(1+q)$  and adding it up with (12), we obtain

$$\begin{aligned} & (1+q) \left( (1+q) - \frac{n-2}{na} \right) \int_M u^{2q} f^2 |\nabla u|^2 + a(1+q) \int_M u^{2(1+q)-\frac{2}{a}} f^2 E \\ & \leq (a(1+q) - \delta) \int_M u^{2(1+q)+\frac{2}{a}} f^2 + \int_M u^{2(1+q)} |\nabla f|^2 + \epsilon \delta \int_M u^{2(1+q)} f^2 \end{aligned} \tag{22}$$

If  $(1+q) a = \frac{n-2}{n}$ , from (22) we obtain

$$\frac{n-2}{n} \int_M u^{2(1+q)-\frac{2}{a}} f^2 E \leq \int_M u^2 |\nabla f|^2 + \epsilon \delta \int_M u^2 f^2 \tag{23}$$

Taking  $f$  as before, we have from (2.23)

$$\frac{n-2}{n} \int_{B(R)} u^{2(1+q)-\frac{2}{a}} E \leq \frac{4}{R^2} \int_{B(2R) \setminus B(R)} u^2 + \epsilon \delta \int_{B(2R)} u^2 \tag{24}$$

If  $|A| > 0$ , let  $\epsilon \rightarrow 0$  and then  $R \rightarrow +\infty$ , by assumption that  $\lim_{R \rightarrow \infty} \frac{1}{R^2} \int_{B(2R) \setminus B(R)} |A|^{\frac{2(n-2)}{n}} = 0$ . From (24) we

conclude that  $E=0$ . Then we obtain  $\sum_{i,j} (h_{ij}^\alpha)^2, \alpha = n+1, n+2, \dots, n+p$  at least have  $p-1$  zeros by (6). So it is easy to



see from a theorem of [5] that  $M$  lies in a totally geodesic  $\mathbb{R}^{n+1} \rightarrow \mathbb{R}^{n+p}$ . Hence  $M$  is an  $\frac{n-2}{n}$  stable complete minimal hypersurface in  $\mathbb{R}^{n+1}$ . Therefore we obtain  $M$  is a catenoid in some Euclidean subspace or an affine plane according to Theorem 3.1 in [11].

## CONCLUSION

We have mathematically discussed the origin and topology of the BIS manifold in four dimensions. Successfully, the conditions under which the BIS manifold becomes hyperplane or affine plane have been derived. Using Cauchy-Schwarz's inequality the structure of the BIS manifold has been designed and modified. Afterwards, using  $\delta$  super-stability inequality the statistical processes occurring in the BIS space have been discussed. The role played by the BIS momenta on the macroscopic scales has been asserted qualitatively and quantitatively.

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