FULLY DEVELOPED MIXED CONVECTION FLOW OF WATER AT 4°C BETWEEN TWO LONG VERTICAL PARALLEL PLATES

Dubewar A.V. and Pandey H. S.

Lokmanya Tilak College of Engineering, Koparkhairane, Navi Mumbai (400 709), India.

ABSTRACT

Numerical results are presented for the fully developed mixed convection flow of water at 4°C between two long vertical parallel plates by modifying the governing equations for flow of water at 4°C without bringing Prandtl number in the picture. Velocity, pressure gradients and bulk temperature profiles are shown on graphs and the effects of different parameters like Gr/Re (buoyancy parameter), r_T (wall temperature difference ratio) on the flow of water are discussed.

KEY WORDS: Mixed convection, vertical plate, flow of water at 4°C, bulk temperature.

INTRODUCTION

Fully developed mixed convection flow between two long vertical parallel plates is studied because it has good practical applications in the industry. It was observed experimentally by Sparrow *et.al* (1984) that flow reversal takes place near the wall where temperature is maintained at a lower level as compared to the temperature of the other wall. In all these studies, the fluid considered was air or water at normal temperature 20°C and in this case, it was observed that the variation of density is a linear function of temperature, viz. $\nabla\rho = \rho\beta(T - T_\infty)$ where T is the temperature, ρ the density and β is the coefficient of volume expansion. However, when the temperature of water falls to 4°C, it was observed that the density of water is maximum and at this temperature, the density does not vary linearly. The variation of density ρ is adequately represented by $\nabla\rho = \rho\gamma (T - T_\infty)^2$ where γ is the thermal expansion coefficient and is given by $\gamma = 0.8 \times 10^{-5}$ (^{o}C)$^{-2}$. This is accurate when T $\approx$ 4°C. So Goren (1966) studied the free convection flow of water at 4°C past a semi-infinite vertical plate. Soundalgekar (1973) established the existence of similarity solution of free convection flow of water at 4°C in the presence of a variable wall temperature. The first theoretical study of fully-developed mixed convection flow was published by Ostrach (1954). Later on, Lietzke (1954), Cebeci *et.al* (1982) studied different types of mixed convection flow between two long vertical parallel plates. However, Aung and Worku (1986) presented the systematic study of such a flow wherein they showed the conditions under which flow reversal near the cooled plate occurs. As viscous dissipative heat was neglected by Aung and Worku (1986), in their study, the temperature viz., Prandtl number, did not appear explicitly. So it is very difficult to predict the flow reversal in terms of Prandtl number. Aung and Worku's case is application for fluids at normal room temperature of 20°C. If the temperature of room falls to 4°C, how the flow of water at 4°C through two vertical long parallel plates is modified? This is an important topic which is not studied in the literature. Hence we now devote this paper to study this aspect. As the governing equations are modified for flow of water at 4°C, without bringing the Prandtl number in the picture, we can study the behaviour of mixed convection flow of water at 4°C. In Section 2, the mathematical analysis is presented and in Section 3, the conclusions are stated.

MATHEMATICAL ANALYSIS

Consider the flow of water at 4°C between two long vertical parallel plates. The *x*-axis is taken along the left-side vertical plate in the upward direction and the *y*-axis is taken normal to the plate. The other plate is held at $y = b$. Let T_1 and T_2 be the temperatures of left and right plate respectively and T_0 is the temperature of the fluid at the entrance of the channel which is 4°C. Then the fully developed flow of water at 4°C can be shown to be governed by

$$O = -\frac{dp}{dx} + \mu \frac{d^2u}{dy^2} + \rho\, g\gamma\, (T - T_\infty)^2 \qquad ..(1)$$

$$O = \frac{d^2T}{dy^2} \qquad ..(2)$$

The boundary conditions are

$$\left.\begin{array}{ll} \text{at } y = 0. & u = 0, \quad T = T_1 \\ \text{at } y \to b, & u = 0, \quad T = T_2 \end{array}\right\} \qquad ..(3)$$

All the physical variables are defined in the Nomenclature.

If u_0 is the velocity of water at the entrance, we now define the following non-dimensional quantities:

$$P = \frac{p}{\rho\, u_0^2}, \quad U = \frac{u}{u_0}, \quad Y = \frac{y}{b}, \quad X = \frac{x}{b\,\mathrm{Re}},$$

$$\left. \mathrm{Re} = \frac{u_0 b}{\nu}, \quad \theta = \frac{T - T_0}{T_2 - T_0}, \quad r_T = \frac{T_1 - T_0}{T_2 - T_0}, \right\} \qquad ..(4)$$

$$Gr = \frac{g\gamma\,(T_2 - T_0)^2\, b}{\nu^2}$$

Substituting eqns. (4) into eqns. (1) -(3), we have

$$-\frac{dP}{dX} + \frac{d^2 U}{dY^2} + \frac{Gr}{\mathrm{Re}}\theta^2 = 0 \qquad ..(5)$$

$$\frac{d^2\theta}{dY^2} = 0 \qquad ..(6)$$

with the following boundary conditions:

$$\left. \begin{array}{llll} Y = 0, & U = 0, & \theta = r_T \\ Y = 1, & U = 0, & \theta = 1 \end{array} \right\} \qquad ..(7)$$

The solution of eqn. (6) satisfying conditions (7) is

$$\theta = (1 - r_T)Y + r_T \qquad ..(8)$$

The solution of eqn.(5) on taking into account (7) and (8) is now given by

$$U = -\frac{Gr}{\mathrm{Re}}\left[\frac{1}{12}Y^4 + \frac{1}{3}r_T(1 - r_T)Y^3 + \frac{1}{2}r_T^2 Y^2\right]$$

$$+ \frac{Gr}{\mathrm{Re}}\left[\frac{1}{12}(1 - r_T)^2 + \frac{1}{3}r_T(1 - r_T) + \frac{1}{2}r_T^2 - \frac{\alpha}{2}\right]Y + \frac{\alpha}{2}Y^2 \qquad ..(9)$$

where $\quad \alpha = -\dfrac{dP}{dX}$.

To determine α, we assume the conservation of mass at any cross-section in the channel, which is given by

$$\int_0^1 U\,dY = 1 \qquad ..(10)$$

Substituting (9) into eqn. (10) and simplifying, we have

$$\alpha = \frac{Gr}{\mathrm{Re}}\left[\frac{12}{40}(1 - r_T)^2 + r_T(1 - r_T) + r_T^2 - 12\right] \qquad ..(11)$$

We have computed numerical values of α for different values of Gr/Re and r_T and these are shown in Figure 1. We observe from this figure that the pressure-gradient decreases with increasing Gr/Re or r_T.

We have now computed numerical values of U on taking into account the values of α computed from eqns. (11) and these are plotted in Figs. 2 and 3. We observe from Fig. 2 that for $r_T = 0.2$, the flow reversal will take place in the left-half of the channel when Gr/Re increases beyond 100, but in the right half space, the velocity increases with increasing Gr/Re, with maximum velocity not exceedingly 2 at Gr/Re = 100.00. In Fig. 3, the variation of velocity for Gr/Re = 100.00 is shown for different values of r_T . It is seen from this figure that for Gr/Re = 100.00, the velocity increases in the left-half of the channel when r_T increases from 0.2 onward. The reverse-type of flow may occur at Gr/Re = 100.00 when r_T decreases below 0.2. However, in the right-half space, the velocity increases with increasing r_T.
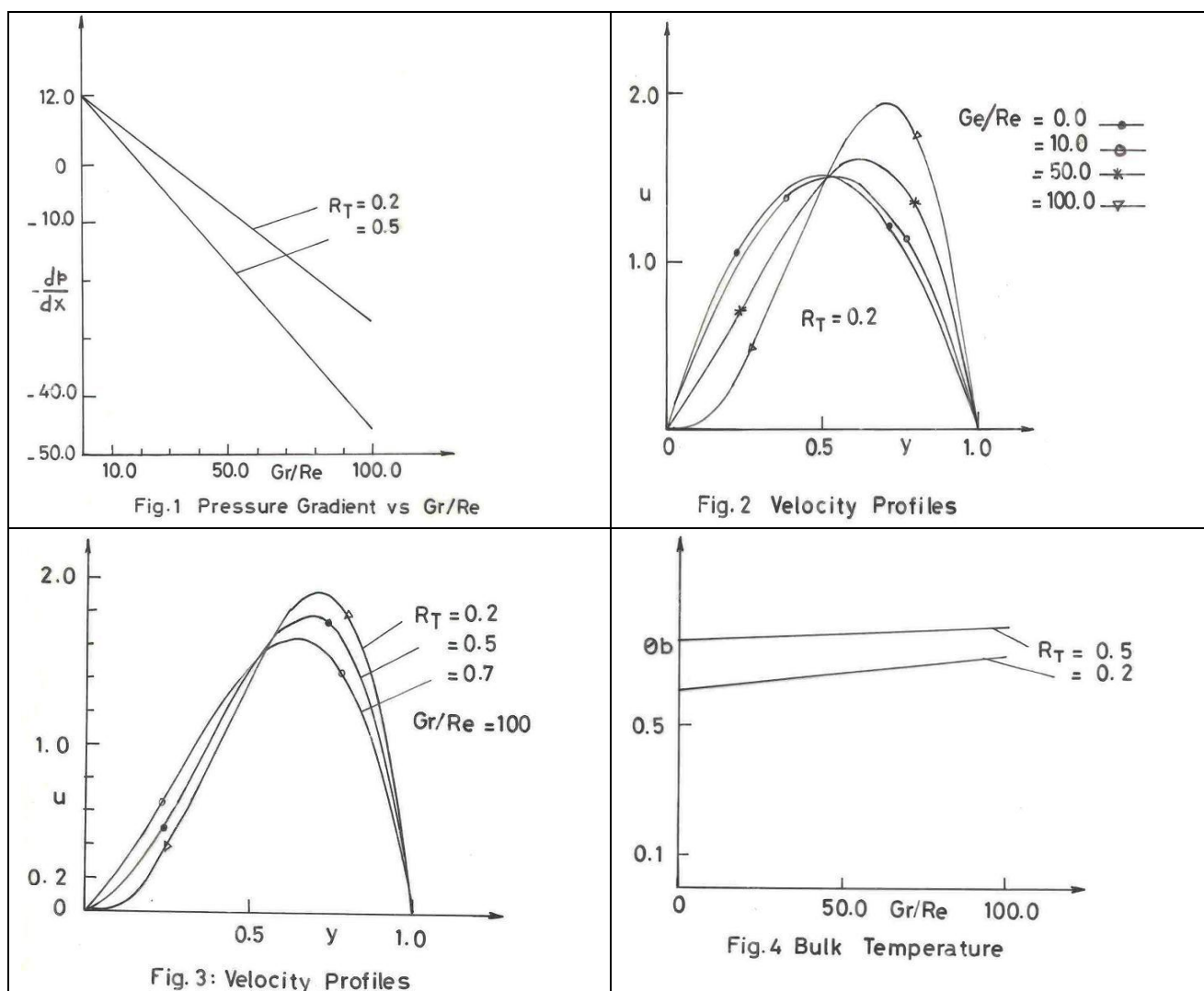
It is interesting to study the effects of these parameters on the bulk temperature θ_b which is given by

$$\theta_b = \frac{\int_0^1 U\theta \, dY}{\int_0^1 U dY} \quad\quad ..(12)$$

Substituting for U and θ, we have carried out the integration and it is given by

$$\theta_b = \frac{(1-r_T)\left[\frac{1}{72}\frac{Gr}{Re}(1-r_T)^2 + \frac{2}{45}\frac{Gr}{Re}r_T(1-r_T) + \frac{1}{24}\frac{Gr}{Re}(r_T)^2 - \frac{\alpha}{24}\right]}{\left[\frac{1}{40}\frac{Gr}{Re}(1-r_T)^2 + \frac{1}{12}\frac{Gr}{Re}r_T(1-r_T) + \frac{1}{12}\frac{Gr}{Re}(r_T)^2 - \frac{\alpha}{12}\right]} \quad ..(13)$$

The numerical values of θ_b are evaluated and these are plotted on Fig. 4. We observe from this figure that the bulk-temperature increases with increasing r_T but it is not significantly affected by Gr/Re.

Fig.1 Pressure Gradient vs Gr/Re

Fig. 2 Velocity Profiles

Fig. 3: Velocity Profiles

Fig. 4 Bulk Temperature

CONCLUSIONS

1. The pressure gradient decreases with increasing Gr/Re or r_T.
2. For $r_T = 0.2$ the flow reversal will take place in the left half of the channel when Gr/Re increases beyond 100, but in the right-half space, the velocity increases with increasing Gr/Re.
3. For Gr/Re = 100.00 the velocity increases in the left half of the channel when r_T increases from 0.2 onward. The reverse type of flow may occur at Gr/Re=100.00 when r_T decreases below 0.2.
4. The bulk-temperature increases with increasing r_T but it is not significantly affected by Gr/Re.

Nomenclature

b - Spacing between plates
β - Coefficient of volume expansion
ρ - Density
γ - Thermal expansion coefficient
$T_1, T2$ -temperature of left and right plate respectively
T_0 - temperature of fluid at the entrance
D_e - hydraulic diameter
g - Acceleration due to gravity
Gr   - Grahoff number
h - Heat transfer coefficient
K - Thermal conductivity
Nu   - Nusselt number
p   - Pressure difference
P - Dimensionless pressure difference
r_T - wall temperature difference ratio
Re   - Reynolds number
T - Temperature
u_0 - velocity of water at the entrance
u - Axial velocity
U - Dimensionless axial velocity
x,y - axial and transverse coordinate, respectively
X - Dimensionless axial coordinate
Y - Dimensionless transverse coordinate
ν - Kinematic viscosity
θ - Dimensionless temperature
θ_b - Bulk temperature

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