ISSN: 2277-5536 (Print); 2277-5641 (Online)

AN EOQ MODEL FOR DETERIORATING ITEMS WITH CONSTANT AND TIME-DEPENDENT DEMAND RATE.

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ABSTRACT
Present study, proposed an inventory model for deteriorating items with constant and time- dependent demand rate under variable rate of deterioration. Deterioration rate is assumed to be constant, increasing and decreasing with time. Numerical example is given to illustrate the model.

KEY WORDS: Deterioration, Replenishment, Time dependent demand.

INTRODUCTION

An inventory model for deteriorating items has engaged attention of researchers in recent years. Ghare and Schrader (1963) developed EOQ model with constant decay. Rafat *et. al*. (1991) established inventory model for deteriorating items with constant demand and finite replenishment rate. Heng *et. al*. (1991) considered a constant demand rate, finite replenishment rate and exponential decay in inventory. Donaldson (1997) proposed inventory model having a linear trend in demand with no shortages. Dave and Patel (1981) developed the inventory model for deteriorating items with linear demand rate with constant decay. Sachan (1984) extended Dave and Patel's model to allow for shortages. Goswami and Chaudhuri (1991), Benkherouf (1995) and Lin *et.al.* (2000) have also established an EOQ models for deteriorating item with shortages and time varying demand. Xu and Wang (1991) and Chung and Ting (2004) studied inventory model with time proportional demand. Kim (1995) developed heuristic model for deteriorating items with linear trend in demand. Chakrabarti and Chaudhuri (1997) studied an EOQ model for deteriorating item having time quadratic demand with shortages. Recently, Srivastava and Gupta (2007) developed an EOQ model for deteriorating item having time dependent demand with constant rate of deterioration. Thus, it is observed that in most of the inventory models are developed with either linear or exponential or quadratic etc. time dependent for whole cycle. But in real situation, demand is constant for some period of time and then it increases or decreases according to the popularity of the product. Actually situation happen when a new product launched in the market. Initially demand of such product is constant for some time and after then, when the product becomes popular in the market, the demand of the product increases or decreases with time. In this research work, developed an inventory model to deal with such type of problem. Here assume that the demand is constant for some period of time and then it increases or decreases with time. It is also assumed that the items do not deteriorate at the beginning of the period. Deterioration of the product starts after some time with an increase or decrease in demand. In this situation, variable rate of deterioration is assumed. Deterioration rate may be constant, increasing and decreasing with time. The rate of deterioration at any time $t > 0$ follow the two parameter Weibull distribution: $Z(t) = \alpha\beta t^{\beta-1}$, where α ($0<\alpha<<1$) is the scale parameter and β(> 0) is the shape parameter. $Z(t)= \alpha$ (rate of deterioration is constant) , when $\beta=1$. Rate of deterioration is increasing with time, when $\beta >1$. Rate of deterioration decreases with time, when $\beta < 1$.

1. Assumptions and Notations

1.1. Assumptions

1. A single item is considered over period of T units of time.
2. The replenishment rate is infinite.
3. Lead time is zero.
4. There is no repair or replacement of deteriorated units.
5. Time horizon of the inventory system is infinite.
6. For the time interval [0. μ], demand is constant at the rate of 'a' unts per unit of time i.e. it doesn't vary with time for this period and for the time interval [μ , T], demand rate is a linear function of time with the form as,
   $$R(t) = a + b(t - \mu), \quad \mu < t \le T .$$
7. The ordering cost, holding cost and unit cost remain constant over time.
8. There is no deterioration for the period [0, μ]. Deterioration rate is follows two parameter Weibull distribution i.e deterioration rate is constant, increasing and decreasing with time for the period [μ , T].

1.2. Notations

T = Length of replenishment cycle.
Q = Number of items received at the beginning of the period.
C_1 = Inventory holding cost per unit per unit of time.
C_2 = unit cost.
C_3 = Set up cost per cycle.
I(t) = On hand inventory level.
μ = The time point at which demand increases with time and also deterioration start.

K(T) = The total cost of the system per unit time.
T* = Optimum value of T.
Q* = Optimum value of Q.
K(T*)= Optimum total cost per unit time.

Mathematical Formulation

The inventory level diminishes due to a constant demand say 'a' units per unit of time during the time interval [0,μ]. After time t=μ, the depletion of the inventory occurs due to effect of demand and deterioration and falls to zero at time t=T.
In time interval [0,μ], demand is constant say 'a' units per unit of time.
Therefore, the total demand in time interval [0,μ] is = aμ
Thus the inventory level reduced by the factor 'aμ' and (q-aμ) inventory is left for the time period [μ,T].
Therefore, the cost of holding inventory for the period [0,μ] is

$$= \frac{C_1}{2}[q+(q-a\mu)]\mu$$

$$= C_1\mu\left[(q-a\mu)+\frac{a\mu}{2}\right]$$

Now, The differential equation that governs the variation of inventory with respect to time is

$$\frac{dI(t)}{dt}+\alpha\beta t^{\beta-1}I(t)=-[a+b(t-\mu)] \quad ; \quad 0\le t\le t_1 \qquad \ldots\ldots(1)$$

Here, t_1= (T-μ), the origin has been shifted just for the sake of mathematical simplicity.
With the boundry conditions
t =0, I(t)= (q-aμ) and t = t_1 ,I(t)=0.
Solution of the differential equation becomes,

$$I(t)e^{\alpha t^{\beta}}-(q-a\mu)=-at-\frac{a\alpha t^{\beta+1}}{\beta+1}-\frac{bt^2}{2}-\frac{b\alpha t^{\beta+2}}{\beta+2}+b\mu t+\frac{b\alpha\mu t^{\beta+1}}{\beta+1}$$

At t =t_1 , I(t) = 0

$$(q-a\mu)=at_1+\frac{a\alpha t_1^{\beta+1}}{\beta+1}+\frac{bt_1^2}{2}+\frac{b\alpha t_1^{\beta+2}}{\beta+2}-b\mu t_1-\frac{b\alpha t_1^{\beta+1}}{\beta+1} \qquad \ldots..(2)$$

The holding inventory cost for the period (0, t_1) is

$$= C_1\,\frac{1}{2}(q-a\mu)t_1 \qquad \ldots..(3)$$

Total amount of inventory that has deteriorates during the cycle is

$$= (q-a\mu)-\int_0^{t_1}[a+b(t-\mu)]dt.$$

$$= \frac{a\alpha t_1^{\beta+1}}{\beta+1}+\frac{b\alpha t_1^{\beta+2}}{\beta+2}-\frac{b\alpha\mu t_1^{\beta+1}}{\beta+1} \qquad \ldots\ldots(4)$$

The total cost per unit of inventory system is given by
K(T) = Inventory carrying cost + deterioration cost + set up cost

$$= \frac{1}{T}\left[C_1\mu(q-a\mu)+\frac{C_1}{2}(q-a\mu)t_1+C_2\left(\frac{a\alpha t_1^{\beta+1}}{\beta+1}+\frac{b\alpha t_1^{\beta+2}}{\beta+2}-\frac{b\alpha\mu t_1^{\beta+1}}{\beta+1}\right)+C_3\right]$$

$$= \frac{1}{T}\left\{C_1\left[\mu a t_1+\frac{a\mu\alpha t_1^{\beta+1}}{\beta+1}+\frac{b\mu t_1^2}{2}+\frac{b\mu\alpha t_1^{\beta+2}}{\beta+2}-b\mu^2 t_1-\frac{b\alpha\mu^2 t_1^{\beta+1}}{\beta+1}+2at_1^2+\frac{2a\alpha t_1^{\beta+2}}{\beta+1}\right.\right.$$
$$\left.\left.+\,bt_1^3+\frac{2b\alpha t_1^{\beta+3}}{\beta+2}-2b\mu t_1^2-\frac{2b\mu\alpha t_1^{\beta+2}}{\beta+1}\right]+C_2\left[\frac{a\alpha t_1^{\beta+1}}{\beta+1}+\frac{b\alpha t_1^{\beta+2}}{\beta+2}-\frac{b\alpha\mu t_1^{\beta+1}}{\beta+1}\right]+C_3\right\} \qquad \ldots\ldots\ldots.(5)$$

The necessary condition for K(T) to be minimum is $\frac{dK(T)}{dT}=0$

$$C_1\left\{a\mu+a\mu\alpha(T-\mu)^{\beta}+b\mu(T-\mu)+b\mu\alpha(T-\mu)^{\beta+1}-b\mu^2-b\mu^2\alpha(T-\mu)^{\beta}+4a(T-\mu)+\right.$$
$$\left.\frac{2a\alpha(\beta+2)(T-\mu)^{\beta+1}}{\beta+1}+\frac{2b\alpha(\beta+3)(T-\mu)^{\beta+2}}{\beta+2}-4b\mu(T-\mu)-\frac{2b\mu\alpha(\beta+2)(T-\mu)^{\beta+1}}{\beta+1}\right\}+$$
$$C_2\left\{a\alpha(T-\mu)^{\beta}+b\alpha(T-\mu)^{\beta+1}-b\alpha\mu(T-\mu)^{\beta}\right\}+C_3=0 \qquad \ldots\ldots.(6)$$

The equation (6) is highly nonlinear equation in T. It can be solved numerically by the Newton-Raphson method.
T* will be an optimal solution, provided the following condition is satisfied for T= T*,

$$\frac{d^2K(T)}{dT^2}>0$$

Substituting the value of T=T* in equation (6), The optimum average cost K(T) can also be determined.

Numerical Example:
The proposed model is illustrated by numerical example with following parameter value is considered.
Let ,

C_1= Rs. 0.50/unit/day
C_2= Rs. 15/unit
C_3= Rs. 75
a= 25 units
b= 0.3
μ= 0.5 days
α= 0.4
β= 3

The optimum value of $T= T^*$ is determined using equation (6). Then the optimum order quantity $Q= Q^*$ and optimal total cost $K(T)= K(T^*)$ are calculated using equation (2) and equation (5) respectively.
The computational results shows that the following optimal values are
$T= T^*= 1.98$ days,
The optimum value of $Q= Q^*=52.604$ units
The optimal total cost per day is $K(T)= K(T^*)$ = Rs. 46.285.

CONCLUSION

Present research work, proposed an inventory model for deteriorating items with constant and time dependent demand rate. Deterioration rate is considered as variable i.e. constant, increases or decreases with time. It is conclude that proposed model minimizes the total cost when deterioration assumed as constant, increasing and decreasing with time. It can be applied where demand of product is constant for some time then, when the product becomes popular in the market.

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