ISSN: 2277-5536 (Print); 2277-5641 (Online)

ON FLOW PAST AN IMPULSIVELY STARTED INFINITE VERTICAL PLATE WITH PERIODIC PLATE TEMPERATURE

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ABSTRACT
An exact solution to the flow past an impulsively started infinite vertical plate is presented when the plate temperature is oscillating about a constant mean. Velocity and temperature profiles are shown *Gr*aphically. It is observed that the velocity increases with increasing the Grashof number but decreases with increasing the frequency ω. Also the skin-friction increases with increasing the frequency ω or the Prandtl number Pr.

KEY WORDS: Flow past, Periodic plate temperature, vertical plate, viscous fluid.

INTRODUCTION

The first exact solution of the Navier-Stokes equation was presented by Stokes (1851) which was concerned with the flow of a viscous incompressible fluid past an impulsively started infinite horizontal plate in a mass of stationary fluid. This is referred in all the textbooks on viscous flow theory. For a semi-infinite horizontal plate started impulsively in a stationary fluid, analytic solution was presented by Stewartson (1951), whereas Hall (1969) solved this problem for semi-infinite horizontal plate by finite-difference method.

If now, an impulsive motion is given to a vertical plate, held stationary in a viscous incompressible fluid, how the flow is affected by free-convection currents in the fluid near the moving plate? This was first presented by Soundalgekar (1977) who gave an exact solution to this problem governed by coupled linear equations. The effect of heating or cooling of the plate by free convection current was also studied. Here the plate temperature was assumed to be isothermal. However, this is a very restricted assumption and in nature, the temperature of the plate cannot remain constant due to many physical reasons. If now, the plate temperature starts oscillating about the mean temperature, how the flow is affected by this new physical phenomenon? This is not studied in the literature even though this is not important aspect of this problem, which is useful in the industry. Hence, it is now proposed to study the effect of oscillating plate temperature on the flow past an impulsively started infinite vertical plate held stationary in an infinite mass of viscous fluid.

Mathematical Analysis

Consider an infinite vertical plate held stationary in a mass of viscous incompressible fluid, both being maintained at the same temperature T'_∞ at $t' \le 0$. At time $t' > 0$, *the* plate is given an impulsive motion with a velocity u_o in the vertically upward direction, the plate temperature raised to T'_w which then starts oscillating with a frequency ω. The y-axis is taken normal to the plate. Then under the usual Boussinesq's approximation, the flow can be shown to be governed by the following set of equations:

$$\frac{\partial u'}{\partial t'} = \nu \frac{\partial^2 u'}{\partial y'^2} + g\beta\left(T' - T'_w\right) \qquad ..(1)$$

$$\rho\, c_p \frac{\partial T'}{\partial t'} = k \frac{\partial^2 T'}{\partial y'^2} \qquad ..(2)$$

with following initial and boundary conditions:

$$\begin{aligned}
&u' = 0, \quad T' = T'_\infty, \quad \text{for all } y', t' \le 0 \\
&\left.\begin{aligned}
&u' = u_o, \quad T' = T'_w + \left(T'_w - T'_\infty\right)\cos\omega' t' \ \text{ at } y' = 0 \\
&u' = 0, \quad T' \to T'_\infty \qquad \text{as } y' \to \infty
\end{aligned}\right\} t' > 0
\end{aligned} \qquad ..(3)$$

Here all the physical quantities are defined in the Nomenclature. As the plate is infinite in extent, the flow variables are functions of y' and t' only.

On introducing the following non-dimensional quantities

$$\left.\begin{aligned}
&u = u'/u_o, \quad y = y'u_o/\nu. \quad t = t'u_0^2/\nu, \quad \omega = \omega'\nu/u_0^2 \\
&\theta = \frac{T' - T'_\infty}{T'_w - T'_\infty}, \quad \Pr = \frac{\mu c_p}{k}, \quad Gr = \frac{\nu\, g\beta\left(T'_w - T'_\infty\right)}{u_0^3}
\end{aligned}\right\} \qquad ..(4)$$

in equations (1) - (3), we have

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + Gr\theta \qquad ..(5)$$

$$\Pr\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} \qquad ..(6)$$

With following initial and boundary conditions:

$$\left.\begin{array}{lll} u=0, & \theta=0 & \text{for all } y, \quad t\le 0 \\ u=1, & \theta=1+\cos\omega t & \text{at } y=0, \\ u=0, & \theta=0 & \text{as } y\to\infty \end{array}\right\} t>0 \qquad ..(7)$$

The solutions to these coupled partial differential equations are derived by the usual Laplace-transform technique and these are as follows:

$$\theta = erfc(\eta\sqrt{\Pr}) + \frac{1}{4}\Big\{e^{-i\omega t}\Big[e^{-2\eta\sqrt{-iwt\Pr}}erfc(\eta\sqrt{\Pr} - \sqrt{-i\omega t})$$
$$+ e^{2\eta\sqrt{-i\omega t\Pr}}.erfc(\eta\sqrt{\Pr} + \sqrt{-i\omega t})\Big] +$$
$$+ e^{i\omega t}\Big[e^{-2\eta\sqrt{i\omega t\Pr}}.erfc(\eta\sqrt{\Pr} - \sqrt{i\omega t}) + e^{2\eta\sqrt{i\omega t\Pr}}erfc(\eta\sqrt{\Pr} + \sqrt{i\omega t})\Big]\Big\} \qquad ..(8)$$

$$u = erfc(\eta) - \frac{4Grt}{(1-\Pr)}i^2 erfc(\eta) -$$
$$-\frac{Gr}{4i\omega(1-\Pr)}\Big\{e^{i\omega t}\Big[e^{-2\eta\sqrt{i\omega t}}.erfc(\eta - \sqrt{i\omega t}) + e^{2\eta\sqrt{i\omega t}}.erfc(\eta + \sqrt{i\omega t})\Big]$$
$$-e^{-i\omega t}\Big[e^{-2\eta\sqrt{-i\omega t}}.erfc(\eta - \sqrt{-i\omega t}) + e^{2\eta\sqrt{-i\omega t}}.erfc(\eta + \sqrt{-i\omega t})\Big]\Big\}$$
$$+\frac{4tGr}{1-\Pr}i^2 erfc(\eta\sqrt{\Pr}) + \frac{Gr}{4i\omega(1-\Pr)}\Big\{e^{i\omega t}\Big[e^{-2\eta\sqrt{i\omega t\Pr}}.erfc(\eta\sqrt{\Pr} - \sqrt{i\omega t}) +$$
$$e^{2\eta\sqrt{i\omega t\Pr}}.erfc(\eta\sqrt{\Pr} + \sqrt{i\omega t})\Big]$$
$$-e^{-i\omega t}\Big[e^{-2\eta\sqrt{-i\omega t\Pr}}.erfc(\eta\sqrt{\Pr} - \sqrt{-i\omega t}) +$$
$$+e^{+2\eta\sqrt{-i\omega t\Pr}}.erfc(\eta\sqrt{\Pr} + \sqrt{-i\omega t})\Big]\Big\} \qquad ..(9)$$

Hence $\eta = y/2\sqrt{t}$, $i^n erfc(X) = -\frac{X}{n}i^{n-1}erfc(X) + \frac{1}{2n}i^{n-2}erfc(X)$

and $i^0 erfc(X) = erfc(X)$. ..(10)

and $erfc(X+iY) = e^{-2iXY}f(X,Y)$

where

$$f(X,Y) = \sum_{n=0}^{\infty}\Big[(XY)^{2\eta}\ \{g_n(x) - i(n+1)g_{n+1}(x)\}\Big]$$

$$g_{n+1}(X) = \frac{2}{2n+1}\left[\frac{e^{-X^2(n+1)}}{X^{2n+1}\sqrt{\pi}} - \frac{g_n(X)}{n+1}\right]$$

$$g_{n+1}(X) = erfc(X)$$

In order to get physical insight into the problem, we have computed numerical values of θ and u , However, it is seen from the expressions for *u,* θ that the argument of *'erfc'*-function is complex and hence we have used formula (10) to separate the real and imaginary parts of *'erfc'* function with complex argument. Pr-values are so chosen that they represent common fluids air (0.71), water (7.0). On Fig. 1, the effect of the *Gr*ashof number and the Prandtl number is shown on the velocity-field and we observe that an increase in *Gr* leads to a raise in the velocity whereas the velocity decreases owing to increasing the Prandtl number. It is seen from the expression for the velocity that the frequency of oscillating plate temperature also affects the velocity and we observe from Fig.2 that the velocity for air and water decrease with increasing the frequency ω but decreases with increasing time *t*, for both air and water. The effect of ω*t* on the velocity field is shown on Fig. 3. We observe from this Figure 3 that the velocity increases with increasing ω*t*.

On Fig. 4, the temperature profiles are shown for different values of ωt and Pr and we observe from this figure that the temperature of both air and water decrease as ωt increases.

Knowing the velocity field, we can now study the skin-friction which is given by

$$\tau = \tau '/\rho u_0^2 = -\frac{1}{2\sqrt{t}}.\frac{du}{d\eta}\bigg|_{\eta=0} \qquad ..(11)$$

Then from (9) and (11), we have

$$2\sqrt{t}\tau = -\frac{2}{\sqrt{\pi}} + \frac{4tGr}{\sqrt{\pi}\left(1+\sqrt{\Pr}\right)} + \frac{Gr}{i\omega\,(1-\Pr)}\left\{e^{i\omega t}.erf\left(\sqrt{i\omega\, t}\right)\left[\sqrt{i\omega\, t} - \sqrt{i\omega\, t\Pr}\right] + e^{-i\omega\, t}.erf\left(\sqrt{-i\omega\, t}\right)\left[\sqrt{-i\omega\, t\Pr} - \sqrt{-i\omega\, t}\right]\right\} \qquad ..(12)$$

The numerical values of τ are computed and are listed in Table I.

Table I - Values of $2\sqrt{t}.\tau$

t	Pr	*Gr*	ωt\ω	5	10	15
0.2	0.71	2	π/2 π/3 π/4 π/6	0.3274 0.2904 0.3363 0.4169	0.4644	0.5224
0.2	0.71	4 6	π/3 π/3	-0.1994 -0.8634		
0.4	0.71	2	π/3	-0.0254		
0.2	7.0	2	π/2 π/3 π/4 π/6	0.7235 0.7048 0.7231 0.7688	0.7928	0.8221
0.2	7.0	4 6	π/3 π/3		0.4572 0.1216	
0.4	7.0	2	π/3		0.5452	

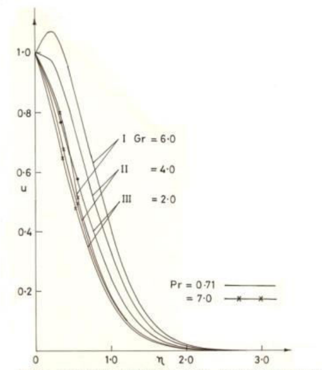

FIG. 1 VELOCITY PROFILES. t=0·2, ω=10·0, ωt = π/3

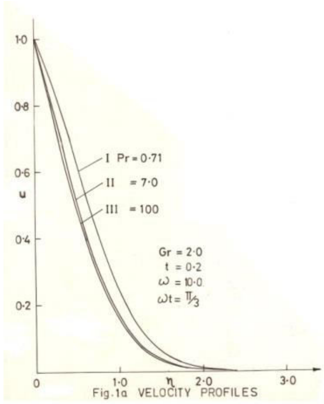

Fig. 1a VELOCITY PROFILES

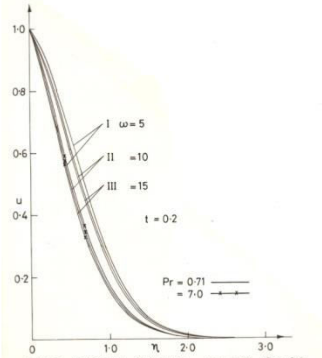

FIG. 2 VELOCITY PROFILES. ωt = π/3, Gr = 2·0

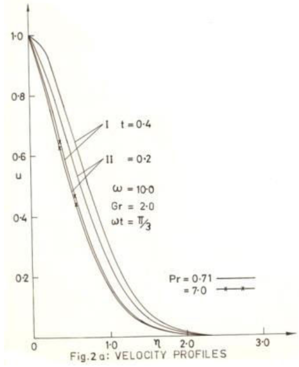

Fig. 2a: VELOCITY PROFILES

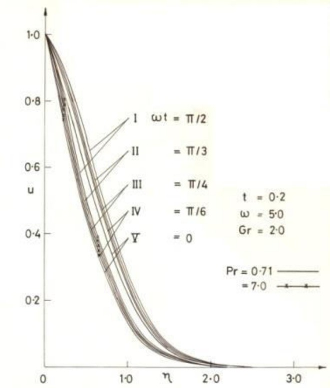

FIG. 3 VELOCITY PROFILES.

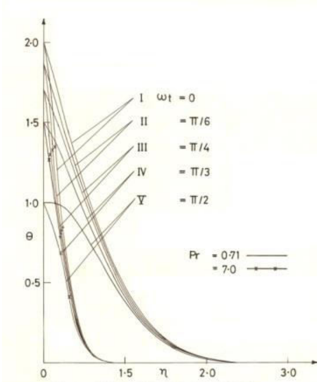

FIG. 4 TEMPERATURE PROFILES.

We observe from this table that at large values of *Gr* or time t , the values of τ for air are negative and hence there may occur separation at the moving plate at large values of *Gr* and small values of the frequency ω. Again, when ωt varied from 30^o to 60^o, the skin-friction decreases. Same effect is observed in water also. But, an increase in Pr, the Prandtl number, or the frequencyω, the skin-friction is also observed to increase.

CONCLUSION

Velocity increases with increasing the *Gr*ashof number *Gr* or time t or ωt, but decreases owing to an increase in the frequency ω or the Prandtl number. Temperature is found to fall with increasing ωt or Pr. At large values of *Gr* and small values of the frequencyω, there may occur separation of air-flow. The skin-friction increases with increasing ω or Pr.

Nomenclature

C_p	specific heat at constant temperature
g	acceleration due to *Gr*avity
Gr	*Gr*ashof number
k	thermal conductivity
Pr	Prandtl number
T'	temperature of fluid near the plate
T'_∞	Temperature of fluid far away from the plate
T'_ω	Plate temperature
t'	time
u_o	impulsive velocity of the plate
u'	velocity of fluid in the upward direction
y'	coordinate normal to the plate
ν	Kinematics viscosity
β	coefficient of volume expansion
ρ	Density
ω'	Frequency
η	$Y/2\sqrt{t}$

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